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## INDUCTIVE REASONING

*by*

Nivedita

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Cognitive functions are incredibly complex, emergent processes that are the essence of being human. One of the many such impressive processes is the ability to perform inductive inferences or inductive reasoning. Without any consciously perceived effort, we generalise from a minimal number of observations and abstract out the essence of one situation and apply it to another. Inductive reasoning plays a vital role everywhere from deriving scientific hypothesis to predicting our day to day activities and preparing for the same.

Induction is an inferential process which evolved to enable us to learn and expand knowledge even in the absence of absolute certainty. Gaining a better perspective and a deeper understanding of induction is central to the psychology of reasoning, to philosophy and the development of artificial intelligence. Some critical questions about induction are

- ‘*What are the concepts and processes that are used in induction?*’
- ‘*How does one transcend from specific observations to general patterns?*’
- ‘*How does induction help to establish connections between completely different domains?*’

Cognitive science tries to find models of different aspects of human cognition, and human logical reasoning is one of the primary disciplines of study. To be able to give a working or computational model for human inductive reasoning will require us to implement the all aspects of human reasoning, including the confidence we have in our conclusions and the ease with which we make them. Induction involves generating new rules and organising new experiences to be referred to in future unfamiliar situations, determining rules which are no longer adequate and replacing them, and refining the rules. The use of analogies across different domains to transfer rules learnt in one to another and having context-free notions which can be applied to varied scenarios are two other modes of induction.

Human reasoning can be very coarsely classified as deductive and inductive. Deductive reasoning involves a collection of statements called syllogisms which includes a finite number of premises followed by a finite number of conclusions. The premises consists of facts. Mathematical logic is the formalisation of deductive reasoning. However, in inductive reasoning, the premises are derived from observation of one or more specific instances in a specific domain, and we generalise from these cases to a more general conclusion or abstraction. Deductive reasoning leads to definite conclusions, whereas probabilistic conclusions are obtained from inductive reasoning.

Philosopher C. S. Peirce [Pei31] posed a fundamental question on the issue of constraints on induction. His main concern was on how humans avoid unnecessarily futile generalisations and forming fruitless hypotheses in the quest for useful generalisations about the nature of things. Peirce claimed that we have a notion of “special aptitude for guessing right and relevant things”, and this points to the fact that the constraints on induction may be innate. Along with the innate structure that guides induction, some constraints can also be obtained solely from the abstraction we make of the human cognitive system as an information processing unit that has goals and receives feedback about the success or failure of its predictions of the complex environment in which it functions.

The problem of induction can be gauged if we somehow begin to quantify the strength of arguments involved in induction. We consider the following factors, representative-ness of observations, and their number and quality. Diversity in the premise promotes stronger induction. For example, let  $P$  be a property of animals.

$$(A) \quad \frac{P(\text{Lions}) \quad P(\text{Mice})}{P(\text{Mammals})}$$

$$(B) \quad \frac{P(\text{Lions}) \quad P(\text{Tigers})}{P(\text{Mammals})}$$

Then (A) is stronger than (B) by the representative-ness conditions. The property about lions and tigers can be generalised better to predators but not to all of the mammals.

Similarly, the effect of number can be illustrated by

$$(C) \quad \frac{P(\text{Lions}) \quad P(\text{Mice}) \quad P(\text{Elephants}) \quad P(\text{Bears})}{P(\text{Mammals})}$$

$$(D) \quad \frac{P(\text{Lions}) \quad P(\text{Bears})}{P(\text{Mammals})}$$

Obviously, (C) is stronger than (D). [HH18]

Rules and rule systems that we obtain from observations are classified into default hierarchies based on inclusion and extrusion relations among the concepts that particular rule system caters to. A set of pointers (different kind of rules) are used to establish the hierarchy between categories.

Variability comes inherent to the representations of a complex system with a default hierarchy. Rule systems which are of the generalisation-exception type are more efficient as compared to the exhaustive stringent rules sans exceptions.

**Example:** Imagine we are in a remote area in the Amazon and we see a bird that the guide calls ‘hungpy’ and it is reddish-orange in colour. The chances are that we will assume the ‘hungpies’ are red-orange. Next, we meet a native of the place from the same tribe as the local guide belongs, and the guide calls him ‘Bimba’ and he is limp. It is highly unlikely that we will assume that ‘Bimbas’ are limp. We now ask the questions:

- ‘What is the cause of the difference in the induction we do?’
- ‘Why is one instance or observation sufficient for a complete induction in some cases, while in others, numerous concurring and congruent instances, without a single exception are needed to establish a universal proposition?’

Mill [Mil43], explored the second question. The answer to this is that humans can store, or can readily construct information about the variability of the kind of object in question concerning the kind of property in question. The number of observations and their quality in terms of reliability, required before we generalise a given observation depends strongly on such variability representations. Furthermore, the induction is stronger if the variability is lower.

To summarise, we pin down the main characteristics of a system capable of induction [HHNT86]. The knowledge of the everyday world, in all its complexity, is represented in rules, organised in categories from elementary to highly abstract in nature. Rules represent relations and clusters of rules (implicit or explicit) act as templates for future inductive mechanisms. New observations are processed in parallel, i.e., the summation of many pieces of evidence is made, complementing each other to create a firm conclusion from weak evidence. There is a default hierarchy in the organisation of representations which accounts for variability (of the environment) and overwriting inadequate rules. Generating predictions is the main goal behind induction, and the underlying mechanisms for induction include the updating parameters such as the strength of existing rules and the creating of possibly useful new rules. Creating new rules is governed by how useful that rule can plausibly be, which is gauged from the success rate of the current predictions we can make. Induction and deduction are not entirely independent as we do refer to deductive rules to make further inductive conclusions.

Generalisations can be of two fundamental kinds: ‘condition- simplifying’ and ‘instance-based’. The former makes the existing rule more general by dropping or ignoring a part of it and in the latter, a new rule is produced associating some existing features of the category. An example of the second kind of generalisation can be the association we learn between being a dog and barking.

The knowledge of the mechanisms of induction can be particularly useful in improving education, where the key goal is learning new rules about the functioning of the world around us. The idea that intuition should be built before formally introducing a subject comes from induction. Along with natural numbers being taught to kids, ideas such as those of ratios, proportions, and fractions can be set in earlier in the education system by game-based learning. Ideas from such learning methods can be further conceptualised. This proposal is based on studies that show young children have the capacity to learn quickly using induction [FH07].

Understanding inductive methods also lay the foundation to understand the advancements and growth of science. This is because in science new laws and theories are proposed, which constitute the highest human inductive achievements.

Analogies are one of the most robust inductive processes. They are bridges between completely different domains and help us deal in an unfamiliar domain by importing experience or rules from the more familiar one [Gar00]. Starting with a few examples, Darwin claimed that his theory of natural selection was motivated by the artificial selection performed by the breeders and farmers; just like breeders modify populations of plants and animals by selecting for traits, so does nature, by making a selection based on the survival. Concepts about the practice of breeding and farming known to humans from ages were imported to a larger domain. Huygens, Young and Fresnel developed the theory that light is a wave by observing the similarities in properties of sound and light which were propagation, reflection etc. Sound was well understood but importing the theory of waves to light was a step made feasible by induction.

Another classic example found in almost all the texts catering to analogies is the ‘Military-Radiation’ problem. This was first used by a gestalt psychologist, Karl Dunker in 1945. A sample was divided into two groups and was told to solve and give their suggestions for a medical problem which was as follows. *The doctor needs to destroy a tumour in a patient’s body with radiation, but a high intensity destroys a lot of healthy tissues with the tumour, and low intensity is insufficient to kill the tumour. The goal is to destroy the tumour without damaging the healthy tissue.* The first group has directly posed the question, whereas the second group was given a source analogue. A story about a military situation was told to them where an army needs to capture a fortress in the centre of a city. Many roads connect the fortress to the outskirts

but are all mined, and although a small group of soldiers could go through each road, if a larger number of troops went, the roads would blast killing all the soldiers. However, the general needs his entire army to capture the fortress, so he comes up with a plan of dividing the troops and sending them all from different roads that reach the fortress, and then launch a successful attack to capture. The solution to the ‘radiation problem’ can be given on similar lines of the military problems by targeting the tumour with low-intensity radiation at different angles so that the healthy tissue on the way does not get damaged, but the radiation focuses on the tumour to destroy it.

Across many experiments, Gick and Holyoak in [GH80] found that about about 75% of participants from the second group generated the convergence solution to the radiation problem after being told the corresponding Military story. In contrast, only about 10% of the participants from the first group could give this solution in the absence of an analogous source.

Various neuro-imaging studies have been conducted to identify the areas of the brain involved in reasoning and whether inductive and deductive reasoning have different centres. Participants were told to comprehend three sentences semantically and in individual trials determine if the third follows from the first two (deductive) or claim that the third is plausible (inductive) given the first two. Simultaneously (as they were performing the task), the brain was PET recorded. It was found that while the deduction cases resulted in activation of the left inferior frontal gyrus, the areas for induction based responses were different. Activation was noted in left medial frontal gyrus, the left cingulate gyrus, and the left superior frontal gyrus [GGKH97].

In Psychology, the most frequently used methods for detecting inductive inferences or confident reasoning in the face of uncertainty are the violation-of-expectation (VOE) tasks. These can be done for both verbal and non-verbal populations, and have many variations which include looking-time tasks and choice-making tasks. Studies were conducted in infants measuring their looking time, under the premise that the higher looking-time indicates there is a violation of expectation, and further inferring that the infant had an expectation based on logic, probability or some other heuristics. If there are 10 yellow balls and 3 blue balls in a toy dispensing machine, which shakes and delivers a toy, we would expect that a yellow ball will pop out if we believe in probabilities. It is surprisingly found to be the same case with infants too, showing they also can use probabilities to make expectations and predictions under uncertain circumstances [DX19].

We have established that inductive reasoning plays a central role in human cognitive functions and deploys many mechanisms:

- Feature attribution involving categorisation of rule systems and generalising the rules for a category after making observations which match the certain pre-established features
- Analogical reasoning mainly focused on extending rules across categories guided by applicability and similarity
- Causal reasoning, although mostly deductive, can have inductive components if we are not given comprehensive information about the causal structure
- Probabilistic judgement based on intrinsic hard-wiring as well as advanced learnt probabilistic techniques

There is a long way to go in fully understanding inductive processes, their development, and their implementation both in humans and in artificial intelligence; and the philosophical questions asking if and when it is right to induct. This quest is bound to carry on in the foreseeable future.

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